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RESEARCH ARTICLE

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Elliptical lobe shape gerotor pump design to minimize wear

Abstract The gerotor pumps are the most important parts of mechanical equipment that have a vast number of applications in industries and automobiles. Because the gerotor pumps cannot be adjusted for wear so it is important to reduce the wear as much as possible. In this paper first mathematical equations for elliptical lobe shape rotors profile and curvature of them have been derived and then Specific flow and wear rate proportional factor (WRPF) have been formulated. To reach the minimum wear in rotors teeth, the ellipse shape factor is changed for each value of number of outer rotor teeth in a feasible range and wear rate proportional factor has been resulted. Also in order to have better comparison specific flow has been presented. The obtained results have been compared with circular pumps with similar geometrical parameters and show the significant improvement in wear of the rotors with negligible changes in the specific flow.

Keywords gerotor pump, elliptical lobe shape pump, wear rate proportional factor (WRPF), specific flow

1 Introduction

An oil pump is an essential part of a hydraulic system. Because of small fluctuation, good performance, high accuracy, compactness and simplicity the gerotor pumps is widely used in the automotive industry for fuel lift, engine oil and transmission systems.

The relevant research on circular lobe shape gerotor pumps includes: Vecchiato et al. [1] developed the geometry of rotor conjugated profiles by using the theory of envelopes to a family of parametric curves and analysis of profile meshing. They also determined singularities of the rotor profiles. Chen et al. [2] developed the correct meshing condition, contact line, contact ratio, calculating method for pin tooth's maximum contact point. Hwang and Hsieh [3] illustrated the use of the envelope theorem for the geometric design of a gerotor pump. They also proposed a mathematical model of the internal cycloidal gear with tooth difference is created by the theory of gearing and determined surface singularities by using the model [4]. Ivanović and Josifović [5] studied specific sliding of trochoidal gearing profile in the gerotor pumps as an element of the wear intensity of the tooth profiles. Chang et al. [6] developed an integrated system for the automated design of a gerotor oil pump. They calculated flow rate and flow ripple irregularity too. Soon Kwon et al. [7] proposed an analytical wear model of a gerotor pump without hydraulic effects namely wear rate proportional factor (WRPF). Gamez-Montero et al. [8] developed an innovative tool to design a trochoidal-gear pump which able to calculate and plot volumetric and kinematic characteristics of the pump. Demenego et al. [9] modified the geometry of rotor profiles of a cycloidal pump which provides only one pair of teeth is in mesh at every instant. It causes to avoid tooth interference and rapid wearing that occur in the case of a conventional pump. Gamez-Montero et al. [10] characterize contact stress of a trochoidal-gear set when it works as part of the hydraulic machine. They verified their model with finite element and prototype model. Kim et al. [11] optimized flow ripple and specific sliding under pressure angle limitation. Inaguma [12] presented theoretical torque (ideal torque) and theoretical displacement in an internal gear pump such as gerotor pump. Maiti and Sinha [13] carried out a kinematic to investigate the pattern of rolling and sliding at the load transmitting (active) contact regions for various type of epitrochoid. Shung and Pennock [14] presented a combined analytical and finite element model for calculating contact forces of a trochoidal machine when friction and deformation at the contact points are neglected. Beard et al. [15] studied the effects of the generating pin size and placement on the curvature and displacement of epitrochoidal gerotors. Mimmi and Pennacchi [16] analyzed internal involute gear pumps and internal lobe pumps that have similar operations but different performances.

Despite of lots of studies carried out on circular lobe

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shape gerotor pumps but there is a few studies related to non-circular gerotor pumps such as: Mimmi and Pennacchi [17] optimized elliptical, sinusoidal, and polycircular lobe shape profiles based on specific performance indexes. They found the elliptical lope pumps have similar flow ripple but smaller specific slipping than circular lobe pumps. Jung et al. [18] developed an automated design system for a rotor with an ellipse lobe profile. They also calculated the flow rate, flow rate irregularity and specific sliding.

2 Geometric design

2.1 Inner rotor profile

Figure 1 shows an elliptical lobe shape gerotor pump. As a gerotor pump the number of teeth of the inner rotor is always one less than outer rotor.

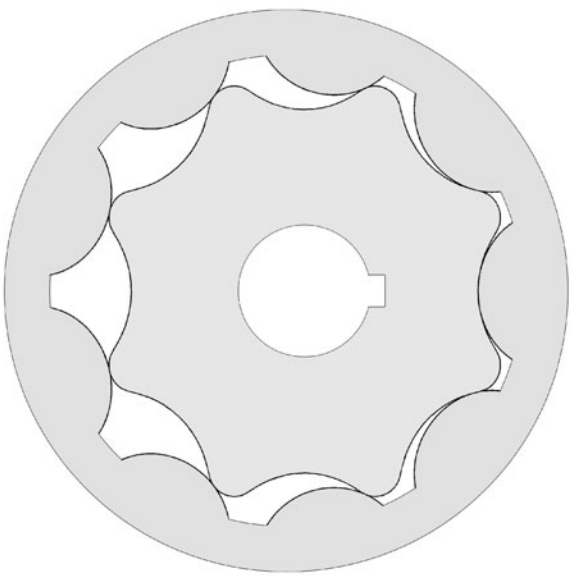

Fig. 1 Elliptical lobe shape pump

It is possible to choose any shape for outer rotor teeth and then generate inner rotor as a conjugate of the outer rotor teeth. So in this paper elliptical teeth has been chosen and the inner rotor has been generated as a conjugate of it.

Consider circles 1 and 2 with radiuses r_1 and r_2 which are in internal tangency (Fig. 2). Point I is the instantaneous center of rotation and e is the eccentric distance between the center of circles 1 and 2. When circle 2 rotates counterclockwise around the circumference of circle 1 in a pure rolling motion, point M generates inner rotor of elliptical lobe shape gerotor. The mathematical equations for generated profile can be derived by using envelope theory.

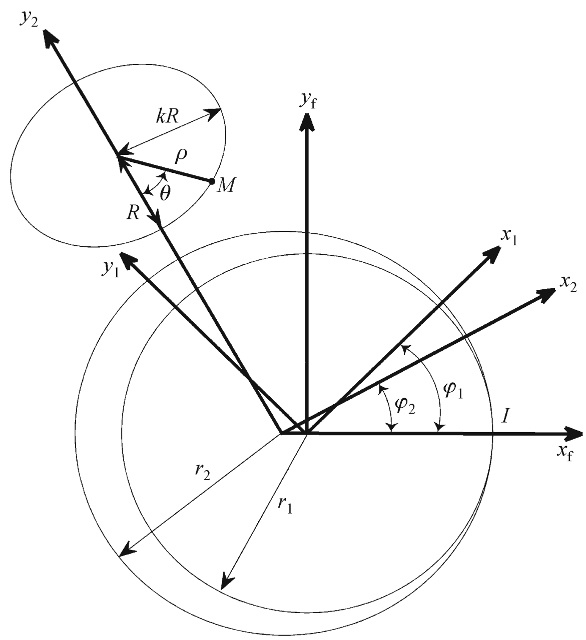

Fig. 2 Coordinate systems and geometrical parameters to generate elliptical lobe shape pumps

As shown in Fig. 2 coordinate systems S_1, S_2 and S_f are connected to the inner rotor, outer rotor and the frame respectively. The rotation angle ratio of the inner and outer rotor is inversely proportional to the ratio of tooth number. Therefore the relationship between the rotation angles φ_1 and φ_2 is represented by the following:

$$\frac{\varphi_2}{\varphi_1} = \frac{N-1}{N}, \tag{1}$$

where N is the number of outer rotor teeth. The coordinate of point M is represented in S_2 as

$$\boldsymbol{r}_2(\xi) = \begin{bmatrix} x_2(\xi) \\ y_2(\xi) \\ 1 \end{bmatrix} = \begin{bmatrix} \rho\sin\theta \\ d-\rho\cos\theta \\ 1 \end{bmatrix} = \begin{bmatrix} kR\sin\xi \\ d-R\cos\xi \\ 1 \end{bmatrix}. \tag{2}$$

where k is the ellipse shape factor and $\tan\theta = k \times \tan\xi$. The normal to profile is formulated as

$$\boldsymbol{N} = \begin{bmatrix} N_x \\ N_y \\ 0 \end{bmatrix} = \frac{\partial \boldsymbol{r}_2}{\partial \xi} \times \boldsymbol{k}, \tag{3}$$

where $\boldsymbol{k}$ is the unit vector in z direction. The equation for the inner rotor can be determined by the following coordinate transformation,

$$\boldsymbol{r}_1(\xi,\varphi_2) = \boldsymbol{M}_{12}(\varphi_2)\boldsymbol{r}_2(\xi), \tag{4}$$

where $\boldsymbol{M}_{12}$ is the transformation matrix from S_2 to S_1 and represented as

$$\boldsymbol{M}_{12}=\begin{bmatrix}\cos(\varphi_2-\varphi_1) & -\sin(\varphi_2-\varphi_1) & -e\sin\varphi\\ \sin(\varphi_2-\varphi_1) & \cos(\varphi_2-\varphi_1) & -e\cos\varphi\\ 0 & 0 & 1\end{bmatrix}. \quad (5)$$

The equation of meshing requires that the normal to the conjugate profiles intersects the instantaneous center of rotation I, whose Cartesian coordinates in S_2 are ($r_2\cos\varphi_2$, $-r_2\sin\varphi_2$) and results

$$f(\xi,\varphi_2)=0 \Leftrightarrow \frac{r_2\cos\ \varphi-x_2}{N_{x2}}=\frac{-r_2\sin\ \varphi-y_2}{N_{y2}}. \quad (6)$$

Substituting Eqs. (2) and (3) in Eq. (6) and simplification yields the meshing rule equation as

$$\begin{aligned}f(\xi,\varphi_2)&=(eN\cos\ \varphi_2-d)\sin\xi\\ &\quad+R(1-k^2)\sin\xi\cos\xi-keN\sin\varphi_2\cos\xi\\ &=0. \quad (7)\end{aligned}$$

Equations (4) and (7) considered simultaneously determine the generated tooth profile of the inner rotor.

2.2 Rotors profile curvature radius

The radius of curvature of a parametric curve can be found by using Eq. (8) [7,17]:

$$\rho=\frac{(x'^2+y'^2)^{3/2}}{x'y''-x''y'}, \quad (8)$$

where x and y are coordinates of the parametric curve, x', y' and x'', y'' are the first and the second derivatives of x and y with respect to the parameter (here is φ_1) respectively.

2.3 Non-undercutting conditions

The undercutting phenomenon occurs in the rotors profile when self-intersecting exists in the curve of the rotor. To avoid undercutting phenomenon the minimum value of the radius of curvature on the convex section of the profile of the rotors should be larger than zero. It is possible to derive explicit formulation for non-undercutting conditions of circular lobe shape profiles but for elliptical ones numerical methods should be used.

3 Specific flow

Specific flow is one of the most important volumetric characteristics of a gerotor pump which should be considered. It is possible to calculate the flow rate of ith chamber of a gerotor pump by using information from the contact points and the inner and outer rotor lobes as in Eq. (9)

$$q_i(\alpha)=\frac{1}{2}H\left[(|O_1A_i|^2-|O_1B_i|^2)-(|O_2A_i|^2-|O_2B_i|^2)\frac{r_2}{r_1}\right]\omega_2, \quad (9)$$

where ω_2 is the outer rotor angular velocity, α is the finite rotation of the outer rotor and H is the rotors width. Other corresponding parameters could be found in Fig. 3.

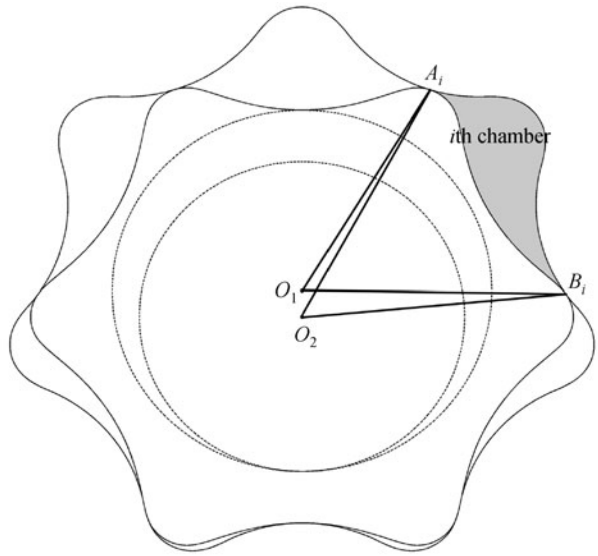

Fig. 3 Corresponding parameters to calculate flow rate for a gerotor pump

Total flow rate for odd and even value of N are presented in Eqs. (10) and (11) respectively.

$$q(\alpha)=\sum_{i=1}^{\frac{N-1}{2}}q_i(\alpha), \quad (10)$$

$$q(\alpha)=\sum_{i=1}^{\frac{N}{2}}q_i(\alpha). \quad (11)$$

The specific flow of a gerotor pump is formulated as

$$Q=(N-1)\int_0^{\frac{2\pi}{N}}q(\alpha)\mathrm{d}\alpha. \quad (12)$$

4 WRPF formulation

Wear in gerotor pump rotors is influenced by sliding velocity and Hertzian contact stress. So to study both sliding velocity and Hertzian contact stress WRPF has been used. WRPF is proportional to the wear rate, between the rotors of the gerotor pump under quasi static and dry contact conditions.

The WRPF is formulated as

$$\mathrm{WRPF} = \frac{P_{\mathrm{H}} V_{\mathrm{s}}}{\omega_1}, \tag{13}$$

where P_{H} is Hertzian contact stress and V_{s} is the sliding velocity. Figure 4 shows needed geometrical parameters for calculating sliding velocity and Hertzian contact stress.

The sliding velocity formula is

$$V_{\mathrm{s}} = \frac{m_i - R}{N}\omega_2, \tag{14}$$

where $m_i = \sqrt{d^2 + (eN)^2 - 2deN\cos\alpha_i}$ and i determine tooth number.

When inner and outer rotors are made of the same material the Hertzian contact stress formula is

$$P_{\mathrm{H}} = \sqrt{\frac{F_i E^*}{2\pi H R^*}}, \tag{15}$$

where $E^* = \dfrac{E}{1-v^2}$, $R^* = (1/\rho_{\mathrm{i}} + 1/\rho_{\mathrm{o}})^{-1}$, E is elastic module, v is poison ratio, ρ_{i} and ρ_{o} is the curvature radius of inner and outer rotor respectively in the ith contact point which is can be calculated by using Eq. (8). F_i is the contact force in the ith contact point which is formulated as

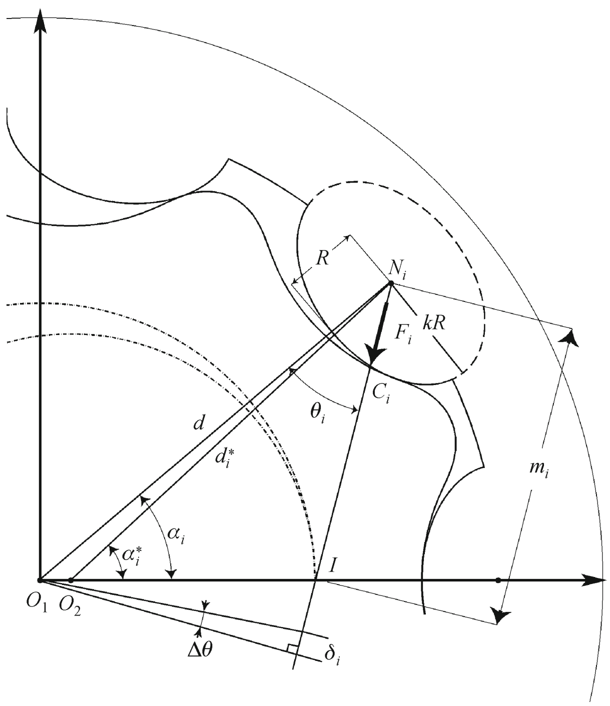

Fig. 4 Geometrical parameters for sliding velocity and Hertzian contact stress calculation

$$F_i = \frac{T l_i^n}{\sum_{j=1}^{N} l_j^{n+1}}, \tag{16}$$

where T is the input torque, $n = (N+1)/N$ and l_i is as follow:

$$l_i = \begin{cases} \dfrac{eNd_i^*}{m_i}\sin\alpha_i^*, & 0 \leqslant \alpha_i < \pi, \\ 0, & \pi \leqslant \alpha_i < 2\pi. \end{cases}$$

5 Numerical results

The main goal of the present paper is perusing the influence of ellipse shape factor on wear rate proportional factor and specific flow. To reach it, all geometrical parameters including d, R and e have been fixed and the specific flow and maximum value of wear rate proportional factor have been calculated for several values of N and k. in other words for each value of N, ellipse shape factor has been increased while the undercutting condition observe. Notice that $E^* = 1$ and $T = 1$ have been used to generalize the rotors material and input torque.

Two commercial pumps with circular lobe shape have been used for comparison. Table 1 shows the geometrical parameters of these pumps.

Table 1 Geometrical parameter of two commercial pumps

	d	R	e	H
Case 1 [7]	40.725	10.85	2.85	9.25
Case 2 (PZ9e19) [8]	22.4	4.55	1.9	8.895

Figure 5 shows WRPF for both cases related to ellipse shape factor for several values of N. For each value of N, there is a k value which minimizes the WRPF. The best value for k depends on the value of the contact force and the radius of curvature of inner and outer rotor in the contact point. For both cases the best value of k decreases for larger values of N because by increasing the number of outer rotor teeth, the radius of inner rotor curvature extremely lessened. For better comparison the improvement in WRPF value of elliptical lobe shape pump than circular one for each value of N has been presented in Table 2.

Whereas the gerotor pumps are designed to provide distinct specific flow so it is necessary to survey specific flow in relation to the ellipse shape factor. As shown in Fig. 6 the specific flow variation in relation to k is slight so by using of the best value of ellipse shape factor WRPF value will be smaller than circular one but the specific flow will not be changed. So the wear in the rotors teeth can be minimized by using an elliptical lobe shape pump instead

Table 2 Improvement in WRPF value of elliptical lobe shape than circular one

		$N=5$	$N=6$	$N=7$	$N=8$	$N=9$	$N=10$
Case 1	k_{best}	2.1	1.5	1.3	1.1	0.8	0.6
	Improvement/%	25.3136	9.2988	3.8603	0.2169	1.8692	14.4050
Case 2	k_{best}	2.4	1.8	1.6	1.5	1.2	
	Improvement/%	32.04	14.55	8.35	3.48	0.91	

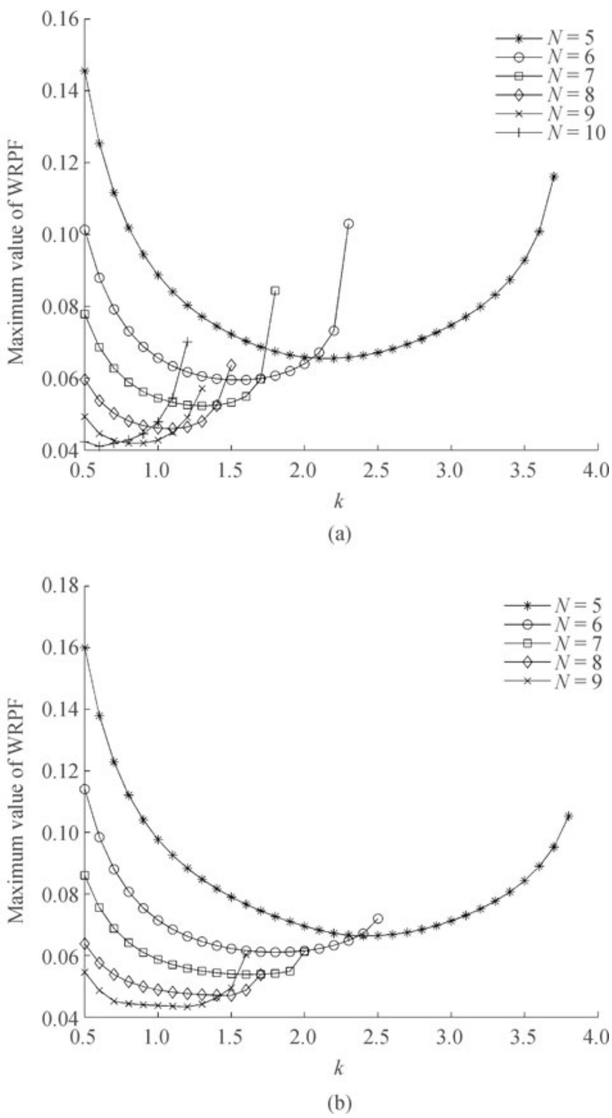

Fig. 5 WRPF vs. ellipse shape factor for several values of N. (a) Case 1; (b) case 2

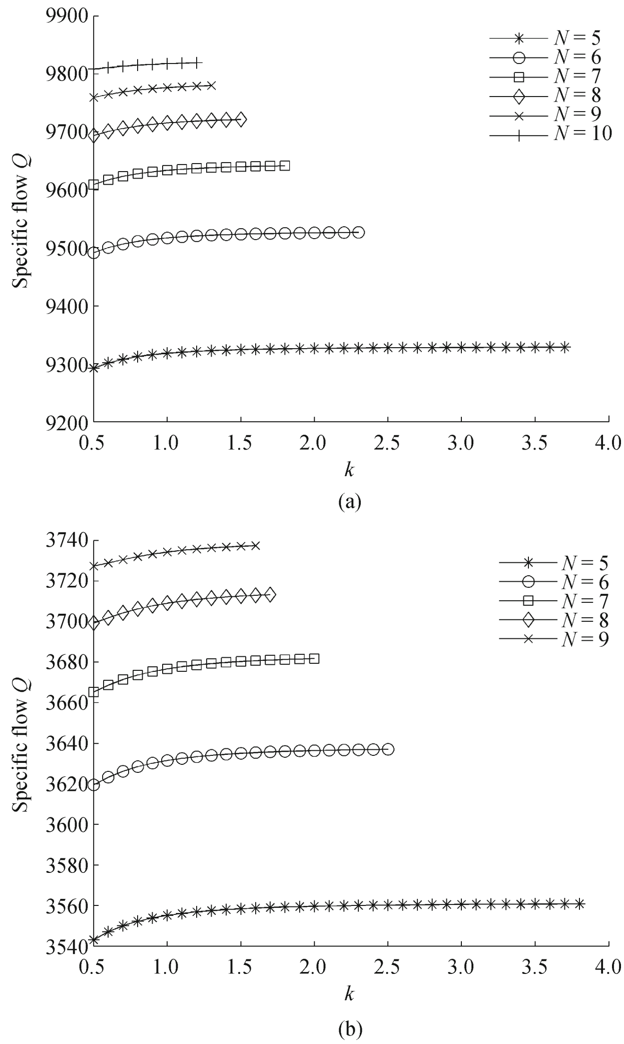

Fig. 6 Specific flow vs. ellipse shape factor for several values of N. (a) Case 1; (b) case 2

of circular one only by changing the ellipse shape factor not other geometrical parameters.

6 Summary and conclusions

Rotors profile equation of elliptical lobe shape pump derived by using envelope theory and then the curvature of the rotors calculated and non-undercutting condition explained. Wear rate proportional factor used to model wear in the rotors teeth and calculated for several values of ellipse shape factor in the feasible range. The obtained results compared with the circular pumps with similar parameters and show significant improvement in the wear of the rotors teeth with negligible changes in the specific flow. So it is possible to reduce wear of the rotors of a gerotor pump by using suitable elliptical lobes.

References

1. Vecchiato D, Demenego A, Argyris J, Litvin F L. Geometry of a cycloidal pump. Computer Methods in Applied Mechanics and Engineering, 2001, 190(18–19): 2309–2330
2. Chen B K, Fang T T, Li C Y, Wang S Y. Gear geometry of cycloid drives. Science in China, Series E: Technology Sciences, 2008, 51 (5): 598–610
3. Hwang Y W, Hsieh C F. Geometry design and analysis for Trochoidal type speed reducers with conjugate envelopes. Transactions of the Canadian Society for Mechanical Engineering, 2006, 30: 261–278
4. Hwang Y W, Hsieh C F. Determination of surface singularities of a cycloidal gear drive with inner meshing. Mathematical and Computer Modeling, 2007, 45(3–4): 340–354
5. Ivanović L, Josifović D. Specific sliding of trochoidal gearing profile in the gerotor pumps. FME Transactions, 2006, 34(3): 121–127
6. Chang Y J, Kim J H, Jeon C H, Kim C, Jung S Y. Development of an integrated system for the automated design of a gerotor oil pump. Journal of Mechanical Design, 2007, 129(10): 1099–1105
7. Kwon S M, Kim M S, Shin J H. Analytical wear model of a gerotor pump without hydrodynamic effect. Journal of Advanced Mechanical Design, Systems, and Manufacturing, 2008, 2: 230–237
8. Gamez-Montero P J, Castilla R, Mujal R, Khamashta M, Codina E. GEROLAB package system: innovative tool to design a trochoidal-gear pump. Journal of Mechanical Design, 2009, 131(7): 074502
9. Demenego A, Vecchiato D, Litvin F L, Nervegna N, Manco S. Design and simulation of meshing of a cycloidal pump. Mechanism and Machine Theory, 2002, 37(3): 311–332
10. Gamez-Montero P J, Castilla R, Khamashta M, Codina E. Contact problems of a trochoidal-gear pump. International Journal of Mechanical Sciences, 2006, 48(12): 1471–1480
11. Kim J H, Kim C, Chang Y J. Optimum design on lobe shapes of gerotor oil pump. Journal of Mechanical Science Technology, 2006, 20(9): 1390–1398
12. Inaguma Y. Calculation of theoretical torque and displacement in an internal gear pump. Koyo Engineering Journal, 2006, 1001E: 70–76
13. Maiti R, Sinha G L. Kinematics of active contact in modified epitrochoid generated rotary piston machines. Mechanism and Machine Theory, 1988, 23(1): 39–45
14. Shung J B, Pennock G R. The direct contact problem in a trochoidal-type machine. Mechanism and Machine Theory, 1994, 29(5): 673–689
15. Beard J E, Yannitell D W, Pennock G R. The effects of the generating pin size and placement on the curvature and displacement of epitrochoidal gerotors. Mechanism and Machine Theory, 1992, 27(4): 373–389
16. Mimmi G C, Pennacchi P E. Involute gear pumps versus lobe pumps: A comparison. Journal of Mechanical Design, 1997, 119(4): 458–465
17. Mimmi G C, Pennacchi P E. Rotor design and optimization in internal lobe pumps. Applied Mechanics Reviews, 1997, 50(11): 133–141
18. Jung S Y, Han S M, Cho H Y, Kim C. Automated design system for a rotor with an ellipse lobe profile. Journal of Mechanical Science Technology, 2009, 23(11): 2928–2937