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Flow irregularity and wear optimization in epitrochoidal gerotor pumps

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Abstract The Gerotor pumps have a vast number of applications in industries and automobiles. The flow rate irregularity and wear rate proportional factor influence on the lifetime of the Gerotor pumps. In this paper, the optimization problem cost functions and constraints have been developed according to volumetric, dynamic and geometric properties. In order to have general optimum solution and reduce number of design variables, all variables have changed to non-dimensional variables by using the outer rotor lobe center radius which causes the non-dimensional cost functions too. The multi-objective optimization problem has been changed to single objective optimization problem by using a multi-objective optimization classical method. The single objective optimization problem has been solved by using of a mixed integer nonlinear optimization algorithm. The optimization programming has been implemented for several values of number of the outer rotor teeth, non-dimensional displacement and rotors width. On the other hand the influence of varied parameter such as number of outer rotors teeth, the value of non-dimensional displacement and non-dimensional rotor width have been surveyed on each of cost functions. The results show in constant input torque, as using greater number of outer rotor teeth up to critical value improves the wear of teeth. Also optimum value of flow rate irregularity in odd values of outer rotor teeth is smaller than even ones. At last for having better comparison, two sample commercial pumps have been optimized. The results show both wear rate proportional factor and flow irregularity have been significantly improved.

Keywords Gerotor pumps · Trochoidal profiles · Flow rate irregularity · Specific flow rate · Wear Rate Proportional Factor · Lifetime optimization

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Nomenclature

N	Number of outer rotor teeth
a	Outer rotor lobe center radius
R	Lobe radius
e	Eccentricity
H	Rotors Width
x_1	Non-dimensional Lobe radius
x_2	Non-dimensional Eccentricity
x_3	Non-dimensional Rotors Width
ω_1	Outer rotor angular velocity
α	Finite rotation of the outer rotor
P_H	Hertzian contact stress
V_s	Sliding velocity
E	Elastic module
ν	Poison ratio
ρ_i	Curvature radius of the i-th contact point
F_i	Contact force in the i-th contact point
T	Input torque
q_w	Requirement displacement
H_{max}	Maximum allowable value of rotors width

1 Introduction

An oil pump is an essential part of a hydraulic system. Because of small fluctuation, good performance, high accuracy, compactness and simplicity the Gerotor pumps is widely used in the automotive industry for fuel lift, engine oil and transmission systems. Whereas the flow rate irregularity and wear of teeth could influence on lifetime of the Gerotor pumps so minimization of them is more required.

The relevant research on Gerotor pumps includes: Daniele Vecchiato et al. [1] developed the geometry of rotor conjugated profiles by using the theory of envelopes to a family of parametric curves and analysis of profile meshing. They also determined singularities of the rotor profiles. CHEN BingKui et al. [2] developed the correct meshing condition, contact line, contact ratio, calculating method for pin tooth's maximum contact point. Yii-Wen Hwang and Chiu-Fan Hsieh [3] illustrated the use of the envelope theorem for the geometric design of a Gerotor pump. They studied both Epitrochoidal and hypotrochoidal Gerotors. In another work Yii-Wen Hwang and Chiu-Fan Hsieh [4] proposed a new mathematical method which can simulate both Gerotors and speed reducers with Epitrochoidal profiles. J.R. Colbourne [5] described for finding the envelopes of Trochoids which perform a planetary motion. The curves have the property that at all times the envelope is in contact with the trochoid at several points, while the gaps between the envelope and the trochoid vary in size as the curves are rotated, so the shapes are potentially applicable in pumps or engines. Joong-Ho Shin, Soon-Man Kwon [6] proposed a simple and exact approach for the lobe profile design of the cycloid plate gear by means of the principle of the instant velocity center in the general contact mechanism and the homogeneous coordinate transformation. Giovanni C. Mimmi and Paolo E. Pennacchi [7] used a general method to show the analytical condition for avoiding undercutting by the use of the concept of the limit curve. Lozica Ivanović and Danica Josifović [8] studied specific sliding of Trochoidal gearing profile in the Gerotor pumps as an element of the wear intensity of the tooth profiles. Soon Man-Know et al. [9] proposed an analytical wear model of a Gerotor pump without hydraulic effects namely wear rate proportional factor (WRPF). P.J. Gamez-Montero et al. [10] they characterize contact stress of a Trochoidal gear set when it works as part of the hydraulic machine. Alberto Demenego et al. [11] modified the geometry of rotor profiles of a cycloidal pump which provides only one pair of teeth is in mesh at every instant. It causes to avoid tooth interference and rapid wearing that occur in the case of a conventional pump. R. Maiti and G.L. Sinha [12] presented a kinematic analysis to investigate the pattern of rolling and sliding at the load transmitting contact regions which can be used to analysis various type of Epitrochoids. J.B. Shung et al. [13] presented a combined analytical and finite element model for calculating contact forces of a Trochoidal machine when friction and deformation at the contact points are neglected. Y. Inaguma [14] presented theoretical torque (ideal torque) and theoretical displacement in an internal gear pump such as Gerotor pump. P.J. Gamez-Montero et al. [15] developed an innovative tool to design a Trochoidal-gear pump which able to calculate and plot volumetric and kinematic characteristics of the pump. J.E. Beard et al. [16] studied the effects of the generating pin size and placement on the curvature and displacement of Epitrochoidal Gerotors. Sung-Yuen Jung et al. [17] described a theoretical analysis of an internal lobe pump and the development of an integrated automated system for rotor design. The designed system generates an elliptical lobe profile and automatically calculates flow rate and flow rate irregularity according to the lobe profile generated. Y.J. Chang et al. [18] developed an integrated system for the automated design of a Gerotor oil pump. They carried out the design optimization to determine the design parameters that maximize specific flow rate and minimize flow rate irregularity. J.H. Kim et al. [19] optimized flow rate irregularity and specific sliding under pressure angle limitation. Giovanni Mimmi and Paolo Pennacchi [20] optimized elliptical, sinusoidal, and Polycircular lobe profiles based on specific performance indexes. They found the elliptical lope pumps have smaller specific slipping than circular lobe pumps. In another work [21] they analyzed internal involute gear pumps and internal lobe pumps that have similar operations but different performances.

According to the literatures it is evident that the optimization of both wear and flow irregularity were not carried out before. However it could be important for design of the Gerotors.

In this paper first independent non-dimensional design variables has been introduced. Then wear rate

Table 2 Non-dimensional Epitrochoidal profile independent design variables for a Gerotor pump

Non-dimensional design variable	x_1	x_2	x_3
Definition	R/a	e/a	H/a

proportional factor (WRPF) and flow irregularity optimization cost functions has been formulated. The non-undercutting condition and specific flow rate have been used to form optimization cost functions.

The most important part of this paper is to choose a suitable optimization method which can be used to optimize both cost functions in the same time. In addition if we would like to consider the number of outer rotor teeth as an optimization design variable, it is necessary to choose a mixed integer nonlinear optimization method. Therefore after checking several optimization algorithms Global Criterion Method [22] has been used for changing multi-objective optimization problem to single one. Then a mixed integer nonlinear optimization algorithm namely Line up Competition Algorithm (LCA) [23–25] has been used to solve the generated problem.

The LCA method was proposed by Yan and was successfully applied to the solution of NLP problems [23] and combinatorial optimization problems [24]. Yan et al. [25] applied this method to solve MINLP problems. They show this method is very effective, robust and reliable.

The selected method could be significantly improved the wear and flow irregularity of the Gerotors which can provide better lifetime.

2 Optimization problem description

2.1 Independent non-dimensional design variables

As shown in Fig. 1, to design Epitrochoidal profiles for a Gerotor pump five independent design variables are needed. These variables are introduced in Table 1.

In order to have general optimum solution and reduce number of design variables, all variables, except number of outer rotor teeth, have changed to non-dimensional variables by using the outer rotor lobe center radius as in Table 2. Notice that a can be valued according to the space where the pump is designed for.

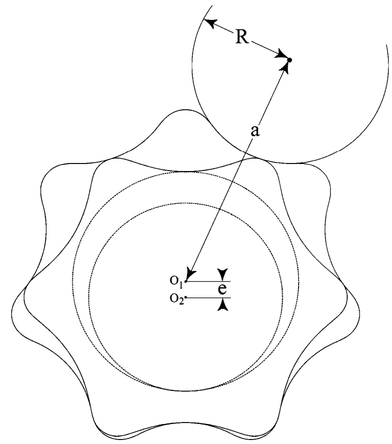

Fig. 1 Epitrochoidal profile independent geometrical design variables for a Gerotor pump

2.2 Flow irregularity cost function

Flow irregularity of a Gerotor pump is an important parameter which influence on lifetime of a Gerotor pump. Larger value for flow irregularity causes more vibration and noise which can damage bearings, shaft and rotors, so minimization of the flow irregularity can improve the lifetime and also performance of a Gerotor pump. The flow rate of each close chamber of a Gerotor pump can be calculated as (1) [15–20]:

$$\overline{q_i}(\alpha) = \frac{1}{2}H\bigg[(|O_1A_i|^2 - |O_1B_i|^2) - (|O_2A_i|^2 - |O_2B_i|^2)\frac{r_2}{r_1}\bigg]\omega_1 \tag{1}$$

where $r_2 = e(N-1)$, $r_1 = eN$, ω_1 is the outer rotor angular velocity and α is the finite rotation of the outer rotor. Other corresponding parameters could be found in Fig. 2.

Total flow rate are presented in (2).

$$\overline{q}(\alpha) = \sum_{i=1}^{M} \overline{q_i}(\alpha) \tag{2}$$

where $M = (N-1)/2$ for odd value of N and $M = N/2$ for even value of N.

Flow irregularity for a Gerotor pump is defined as (3):

$$FR = \frac{\max(\overline{q}(\alpha)) - \min(\overline{q}(\alpha))}{average(\overline{q}(\alpha))} \tag{3}$$

Table 1 Epitrochoidal profiles independent design variables for a Gerotor pump

Design variable	N	a	R	e	H
Description	Number of outer rotor teeth	Outer rotor lobe center radius	Lobe radius	Eccentricity	Rotors Width

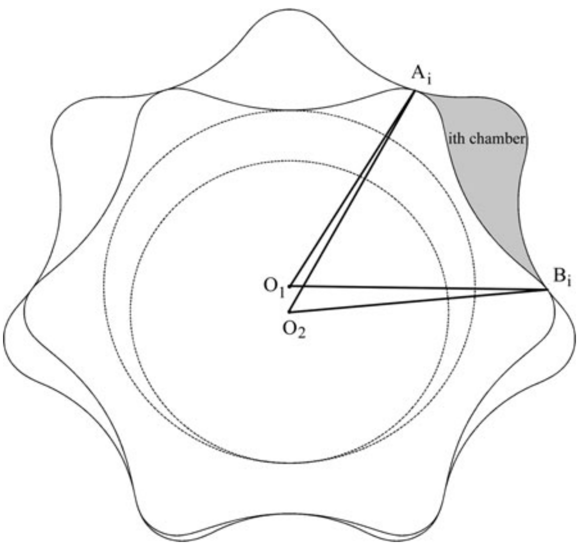

Fig. 2 Corresponding parameters to calculate flow rate for a Gerotor pump

So the first cost function could be formulated as in (4):

$$f_1(N, x_1, x_2, x_3) = FR(N, a, R, e, H) = FR(N, 1, x_1, x_2, x_3) \quad (4)$$

2.3 Wear Rate Proportional Factor cost function

Wearing in Gerotor pump rotors is influenced by sliding velocity and Hertzian contact stress. So to study both sliding velocity and Hertzian contact stress "Wear Rate Proportional Factor (WRPF)" has been used. WRPF is proportional to the wear rate, between the rotors of the Gerotor pump under quasi static and dry contact conditions [9].

The WRPF is formulated as (5) [9]:

$$WRPF = \frac{P_H V_s}{\omega_1} \quad (5)$$

where P_H is Hertzian contact stress and V_s is sliding velocity.

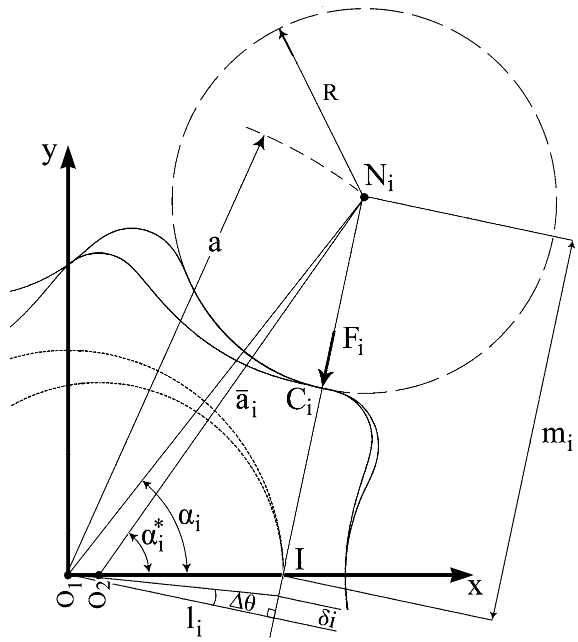

Fig. 3 Geometrical parameters for sliding velocity and WRPF calculation

Figure 3 shows geometrical parameters for calculating sliding velocity and Hertzian contact stress. The sliding velocity formula is as follow:

$$V_s = \frac{m_i - R}{N}\omega_1 \quad (6)$$

where $m_i = \sqrt{a^2 + (eN)^2 - 2aeN\cos(\alpha_i)}$. Notice that i determine tooth number.

When inner and outer rotors are made of the same material the Hertzian contact stress formula is as follow:

$$P_H = \sqrt{\frac{F_i E^*}{2\pi H R^*}} \quad (7)$$

where $E^* = \frac{E}{1-\upsilon^2}$, $R^* = (1/\rho_i + 1/R)^{-1}$, E is elastic module, ν is poison ratio, ρ_i is the curvature radius of the i-th contact point which is formulated as (8) and

F_i is contact force in the i-th contact point which is formulated as (9).

$$\rho_i = a\frac{\sqrt[3]{(Ne)^2 + a^2 - 2eNa\cos(\varphi_i)}}{-N^3e^2 - a^2 + eN(N+1)a\cos(\varphi_i)} + R \quad (8)$$

$$F_i = \frac{Tl_i^n}{\sum_{j=1}^N l_j^{n+1}} \quad (9)$$

where φ_i is the angle between O_2C_i and $\mathbf{x}$ axis, T is the input torque, $n = \frac{N+1}{N}$ and l_i is as follow:

$$l_i = \begin{cases} \frac{e(N-1)\bar{a}_i}{m_i}\sin(\alpha_i^*) & 0 \le \alpha_i < \pi \\ 0 & \pi \le \alpha_i < 2\pi \end{cases}$$

So the second cost function could be formulated as (10).

$$\begin{aligned} f_2(N, x_1, x_2, x_3) &= \frac{WRPF(N, a, R, e, H)}{\sqrt{TE^*}} \\ &= \frac{WRPF((N, 1, x_1, x_2, x_3))}{\sqrt{TE^*}} \end{aligned} \quad (10)$$

Notice that *WRPF* depends on E, ν and input torque (T) but the optimum point does not depend on them, so the second cost function is defined as in (10).

2.4 Constraints

2.4.1 Non-undercutting constraint

The undercutting phenomena occurs in the rotor profile when points with zero radius of curvature are observed, so to avoid this phenomena the minimum value of the radius of curvature of the tooth profile should not be less than zero. The non-undercutting condition can be explained as follow [9]:

$$\frac{\sqrt{27(N-1)(a^2 - e^2N^2)}}{(N+1)^{3/2}} \ge R \quad (11)$$

2.4.2 Displacement constraint

Gerotor pumps are categorized as positive displacement pumps and the design of them is performed according to the requirement positive displacement. The positive displacement of a Gerotor pump is formulated as in (12) [15–20]:

$$Q(N, a, R, e, H) = (N-1)\int_0^{\frac{2\pi}{N}} \overline{q}(\alpha)d\alpha \quad (12)$$

Non-dimensional displacement is defined as:

$$\begin{aligned} q &= q(N, x_1, x_2, x_3) \\ &= \frac{Q(N, a, R, e, H)}{a^3} = Q(N, 1, x_1, x_2, x_3) \end{aligned} \quad (13)$$

So the displacement constraint can be formulated as:

$$q = q_w \quad (14)$$

where q_w is the requirement displacement and defined by the user.

2.4.3 Boundary constraints

Using the non-undercutting condition leads the boundary constraints for x_1 and x_2. Equation (11) leads (15) as:

$$x_2 \le \frac{1}{N}\sqrt{1 - \frac{x_1(N+1)^3}{27(N-1)}} \quad (15)$$

So two boundary constraints can be obtain by using (15):

$$0 < x_1 \le \frac{27(N-1)}{(N+1)^3} \quad (16)$$

$$0 < x_2 \le \frac{1}{N} \quad (17)$$

Definition of the boundary constraints limits the solution space and speed up the optimization convergence. Last boundary constraint can be formulated by attention to the rotors width as:

$$0 < x_3 \le x_{3max} \quad (18)$$

where $x_{3max} = \frac{H_{max}}{a}$ and H_{max} is the maximum value of H which determine according to the space where the Gerotor pump design for.

3 Solution method

3.1 Multi objective solution procedure

There are several classical methods which use to change multi objective optimization problem to single objective one. In this paper "Global Criterion Method" has been used. As shown in Fig. 4, by using of this method, a Pareto point with the minimum distance to

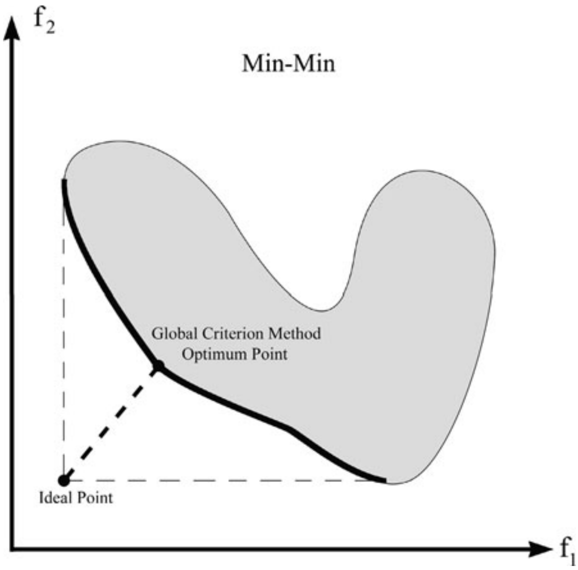

Fig. 4 A scheme of Global Criterion Method for two cost functions

the ideal point is obtained as the optimum solution of the multi objective optimization problem [22]. Since to design a vehicle only a set of design variables are needed so finding which Pareto point is the best is a problem. When there is not any priority between cost functions the GCM can be the most useful method. Also it is possible to allocate a weight to each cost function to consider a priority for them [22].

On the other hand each optimization method is not applicable for all optimization problems but GCM is applied successfully to the problem.

By using of the GCM the optimization problem can be formulated as follow:

$$\min \quad \left[\left(\frac{f_1}{f_1^*}-1\right)^2+\left(\frac{f_2}{f_2^*}-1\right)^2\right]^{1/2}$$

S.t:

$$x_1-\frac{\sqrt{27(N-1)(1-x_2^2N^2)}}{(N+1)^{3/2}}\le 0$$

$$0<x_1\le\frac{27(N-1)}{(N+1)^3}$$

$$0<x_2\le\frac{1}{N}$$

$$0<x_3\le x_{3max}$$

$$q(N,x_1,x_2,x_3)=q_w$$

where f_j^* is the optimum value of the j-th cost function.

3.2 Single objective solution procedure

There are a vast number of optimization methods which involve single objective optimization problems. In this paper line-up competition algorithm (LCA) has been used because the LCA method can be used for non-continues and infinite cost functions and constraints and also is applicable in mixed integer nonlinear problems (MINLP) [23–25]. Notice that for some value of the variables such as $x_1=0$ one or both cost functions might be infinite. Also if we'd like to consider N as an optimization design variable the optimization problem will be a MINLP. We consider N as a design variable to find the value of critical N which will be discussed in Sect. 4.

The LCA is an evolutionary algorithm. In the LCA, all families are independent and parallel during the evolutionary process, each family producing offspring only by the asexual reproduction. Moreover, there are two levels of competition in the algorithm. One is the survival competition inside a family. The best one of every family survives in every generation. The second is the competition between families. According to the values of their objective function, all families are ranked to form a line-up. The best family is in the first position in the line-up, while the worst is in the final position. Through the competitions of the two different means, the first family in the line-up is replaced continually by other families, or the value of objective function of the first position is updated continually. As a result, the optimal solution is approached rapidly [25].

4 Numerical results

4.1 General optimum solution

Tables 3 to 5 show the optimum point and optimum value of the multi objective optimization problem for several values of N and H_{max}. In all cases the optimum value of x_3 is x_{3max}. It is seen that as x_{3max} increases flow irregularity increase for even value of N while it decrease for odd value of N. Also WRPF is decreased by increasing of x_{3max} for both odd and even value of N. Note that increasing number of teeth reduces the ability of providing the non-dimensional displacement

(q_w) in constant non-dimensional rotor width (x_3). In other word by increasing of q_w/x_3 the maximum value of N which can provide the requirement q_w would be decreased (Fig. 5), for example a pump with $N = 8$ and $x_3 = 0.25$ cannot provide $q_w = 0.4$ (Fig. 5) because by increasing the tooth number the chamber volumes are decreased.

Figure 6 shows the optimum value of flow irregularity in comparison to the number of outer rotor teeth for several values of non-dimensional displacement and x_{3max} corresponding to Tables 3 to 5. Odd values for number of outer rotor teeth are causing better optimum values of flow irregularity than even values. In addition larger number of outer rotor teeth decreases the optimum value of the flow irregularity.

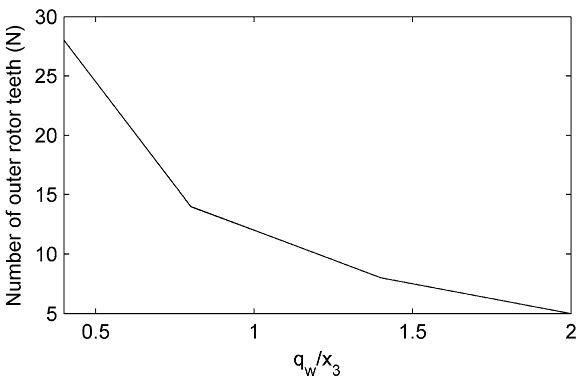

Fig. 5 Maximum number of outer rotor teeth which can provide specific displacement VS q_w/x_3

As shown in Fig. 6, for even values of N optimum value of flow irregularity is decreased by increasing of non-dimensional displacement but it is different for odd values of N.

Figure 7 shows the optimum values of WRPF in comparison to the number of outer rotor teeth for several values of non-dimensional displacement and x_{3max} corresponding to Tables 3 to 5. The optimum value of WRPF is decreased by growth in N. To vindicate this phenomenon it is necessary to tent on the parameters which influence on WRPF. The WRPF is influenced by two parameters which are radius of curvature of rotors profile and the value of contact force in each contact point. As N is increased the denominator of (9) would increased too so the contact force is decreased (in constant input torque). In another word the number of contact points is increased and the input torque is distributed on these points. Increasing of N has another influence which is reduction in radius of curvature of the rotors profile which increases Hertzian contact Stress or WRPF. So let us define a critical value of number of outer rotor teeth (N_{cr}). Assume that $N < N_{cr}$. In this case as N increases the effect of decreasing of contact forces is stronger than the effect of reduction in radius of curvature. The results are reversed for $N > N_{cr}$. Table 6 shows the value of N_{cr} for several values of q_w. These results are obtained by setting up a mixed integer nonlinear optimization problem with only one cost function WRPF.

Table 3 Optimization results for $q_w = 0.1$

N	$H_{max} = 0.25$			$H_{max} = 0.5$			$H_{max} = 1$		
	Optimum point	FR (%)	WRPF	Optimum point	FR (%)	WRPF	Optimum point	FR (%)	WRPF
3	(0.462, 0.0716, 0.25)	18.70	1.708	(0.573, 0.0458, 0.5)	18.78	1.138	(0.679, 0.0301, 1)	18.84	0.549
4	(0.644, 0.0978, 0.25)	25.97	0.835	(0.648, 0.0503, 0.5)	30.45	0.542	(0.648, 0.0253, 1)	31.68	0.313
5	(0.378, 0.0553, 0.25)	6.62	0.669	(0.489, 0.0337, 0.5)	6.55	0.464	(0.500, 0.0172, 1)	5.79	0.294
6	(0.394, 0.0552, 0.25)	12.95	0.520	(0.394, 0.0277, 0.5)	13.63	0.414	(0.394, 0.0139, 1)	13.79	0.269
7	(0.279, 0.0465, 0.25)	3.16	0.480	(0.316, 0.0245, 0.5)	2.91	0.376	(0.316, 0.0123, 1)	2.72	0.245
8	(0.259, 0.0444, 0.25)	6.99	0.403	(0.259, 0.0222, 0.5)	7.24	0.339	(0.259, 0.0111, 1)	7.29	0.223
9	(0.211, 0.0422, 0.25)	1.84	0.364	(0.216, 0.0213, 0.5)	1.68	0.304	(0.216, 0.0106, 1)	1.60	0.201
10	(0.183, 0.0401, 0.25)	4.36	0.332	(0.183, 0.0201, 0.5)	4.50	0.280	(0.183, 0.0101, 1)	4.53	0.185
11	(0.156, 0.0390, 0.25)	1.18	0.304	(0.156, 0.0195, 0.5)	1.10	0.258	(0.156, 0.0097, 1)	1.06	0.1697
12	(0.135, 0.0375, 0.25)	3.27	0.281	(0.135, 0.0187, 0.5)	3.37	0.239	(0.135, 0.0094, 1)	3.39	0.157
13	(0.118, 0.0367, 0.25)	0.82	0.262	(0.118, 0.0183, 0.5)	0.77	0.222	(0.118, 0.0092, 1)	0.75	0.146
14	(0.104, 0.0365, 0.25)	2.33	0.244	(0.104, 0.0183, 0.5)	2.41	0.206	(0.104, 0.0091, 1)	2.43	0.135
15	(0.0923, 0.0369, 0.25)	0.61	0.228	(0.0923, 0.0185, 0.5)	0.57	0.192	(0.0923, 0.0093, 1)	0.56	0.125

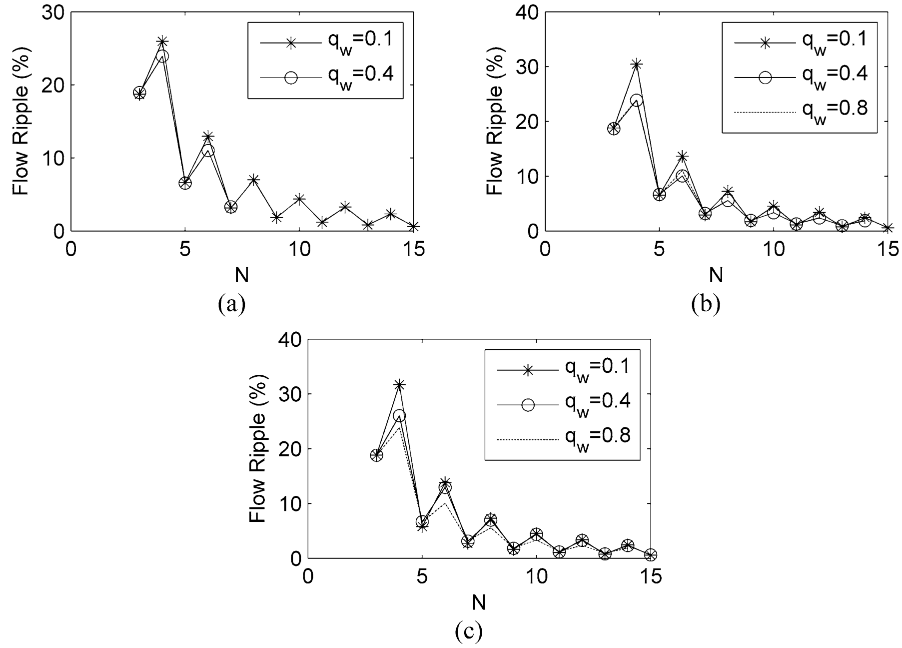

Fig. 6 Optimum value of flow irregularity for various non-dimensional displacements and (**a**) $x_{3max} = 0.25$. (**b**) $x_{3max} = 0.5$. (**c**) $x_{3max} = 1$ vs. number of outer rotor teeth

Table 4 Optimization results for $q_w = 0.4$

N	$H_{max} = 0.25$			$H_{max} = 0.5$			$H_{max} = 1$		
	Optimum point	FR (%)	WRPF	Optimum point	FR (%)	WRPF	Optimum point	FR (%)	WRPF
3	(0.231, 0.199, 0.25)	18.94	1.568	(0.342, 0.117, 0.5)	18.69	0.938	(0.465, 0.0721, 1)	18.82	0.599
4	(0.348, 0.209, 0.25)	23.93	0.817	(0.525, 0.145, 0.5)	23.87	0.476	(0.643, 0.0975, 1)	26.03	0.294
5	(0.130, 0.157, 0.25)	6.55	0.635	(0.261, 0.0928, 0.5)	6.67	0.354	(0.383, 0.0557, 1)	6.68	0.234
6	(0.158, 0.157, 0.25)	11.01	0.473	(0.384, 0.107, 0.5)	10.06	0.297	(0.394, 0.0552, 1)	12.95	0.184
7	(0.0595, 0.142, 0.25)	3.30	0.572	(0.171, 0.0808, 0.5)	3.19	0.262	(0.271, 0.0460, 1)	3.12	0.172
8	*****	*****	*****	(0.259, 0.0882, 0.5)	5.56	0.216	(0.259, 0.0444, 1)	6.99	0.142
9	*****	*****	*****	(0.123, 0.076, 0.5)	1.91	0.194	(0.208, 0.0421, 1)	1.83	0.129
10	*****	*****	*****	(0.183, 0.0807, 0.5)	3.32	0.167	(0.183, 0.0401, 1)	4.36	0.117
11	*****	*****	*****	(0.0867, 0.0719, 0.5)	1.28	0.168	(0.156, 0.039, 1)	1.18	0.108
12	*****	*****	*****	(0.133, 0.0745, 0.5)	2.38	0.143	(0.135, 0.0375, 1)	3.27	0.0992
13	*****	*****	*****	(0.0584, 0.0686, 0.5)	0.94	0.151	(0.118, 0.0367, 1)	0.82	0.0925
14	*****	*****	*****	(0.0637, 0.0698, 0.5)	1.87	0.134	(0.104, 0.0365, 1)	2.34	0.0863
15	*****	*****	*****	*****	*****	*****	(0.0923, 0.037, 1)	0.61	0.0805

4.2 Optimization of commercial pumps

In this section two commercial pumps have been chosen, Table 7 shows the geometrical characteristics of them. First the pump displacement is calculated and used to form the equal constraint, and then the optimum solution is found. Two values of N are used for optimization. Table 8 shows the optimum point, optimum value of each cost function and improvement criterion of them. All two cost functions have been improved. Using $N = 11$ has better improvement than

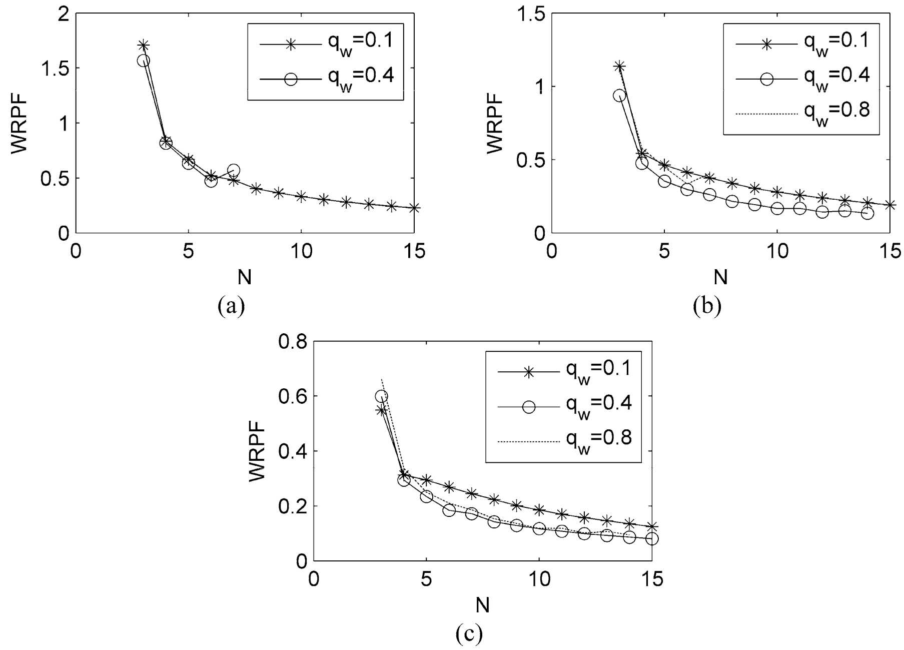

Fig. 7 Optimum value of WRPF for various non-dimensional displacements and (**a**) $x_{3max} = 0.25$. (**b**) $x_{3max} = 0.5$. (**c**) $x_{3max} = 1$ vs. number of outer rotor teeth

Table 5 Optimization results for $q_w = 0.8$

N	$H_{max} = 0.25$			$H_{max} = 0.5$			$H_{max} = 1$		
	Optimum point	FR (%)	WRPF	Optimum point	FR (%)	WRPF	Optimum point	FR (%)	WRPF
3	(0.0791, 0.332, 0.25)	16.73	2.831	(0.230, 0.198, 0.5)	18.91	1.111	(0.343, 0.117, 1)	18.75	0.661
4	*****	*****	*****	(0.347, 0.209, 0.5)	23.96	0.577	(0.525, 0.145, 1)	23.86	0.336
5	*****	*****	*****	(0.130, 0.157, 0.5)	6.55	0.449	(0.260, 0.0927, 1)	6.66	0.250
6	*****	*****	*****	(0.156, 0.157, 0.5)	11.04	0.335	(0.384, 0.107, 1)	10.06	0.210
7	*****	*****	*****	(0.0593, 0.142, 0.5)	3.30	0.405	(0.170, 0.0807, 1)	3.18	0.186
8	*****	*****	*****	*****	*****	*****	(0.259, 0.0882, 1)	5.56	0.153
9	*****	*****	*****	*****	*****	*****	(0.127, 0.0762, 1)	1.93	0.136
10	*****	*****	*****	*****	*****	*****	(0.183, 0.0807, 1)	3.32	0.118
11	*****	*****	*****	*****	*****	*****	(0.0867, 0.0719, 1)	1.28	0.119
12	*****	*****	*****	*****	*****	*****	(0.133, 0.0745, 1)	2.38	0.101
13	*****	*****	*****	*****	*****	*****	(0.0584, 0.0686, 1)	0.94	0.107
14	*****	*****	*****	*****	*****	*****	(0.0637, 0.0698, 1)	1.87	0.0948
15	*****	*****	*****	*****	*****	*****	*****	*****	*****

$N = 9$. Figure 8 shows the profile of Case 1 and its optimum profiles for example. The “Wear Rate Proportional Factor” and “Flow Rate” diagrams for case 1 and its optimum profiles are depicted in Figs. 9 and 10 respectively to have better comparisons.

5 Conclusion

In this paper two cost functions which influence on the lifetime of a Gerotor pump have been introduced and formulated. The geometrical and volumetric con-

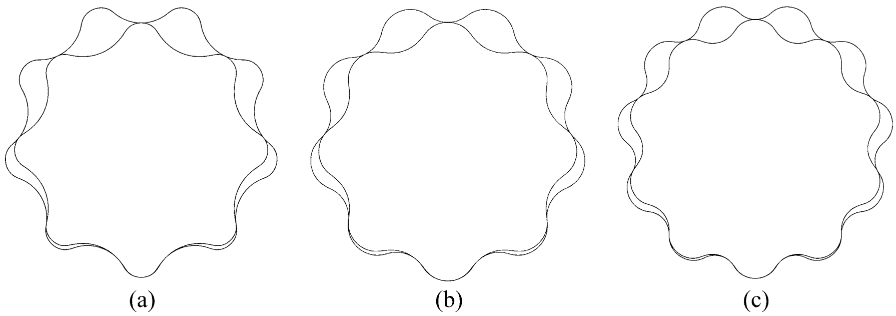

Fig. 8 (**a**) Case 1 profile, (**b**) optimum profile of Case 1 with $N = 9$, (**c**) optimum profile of Case 1 with $N = 11$

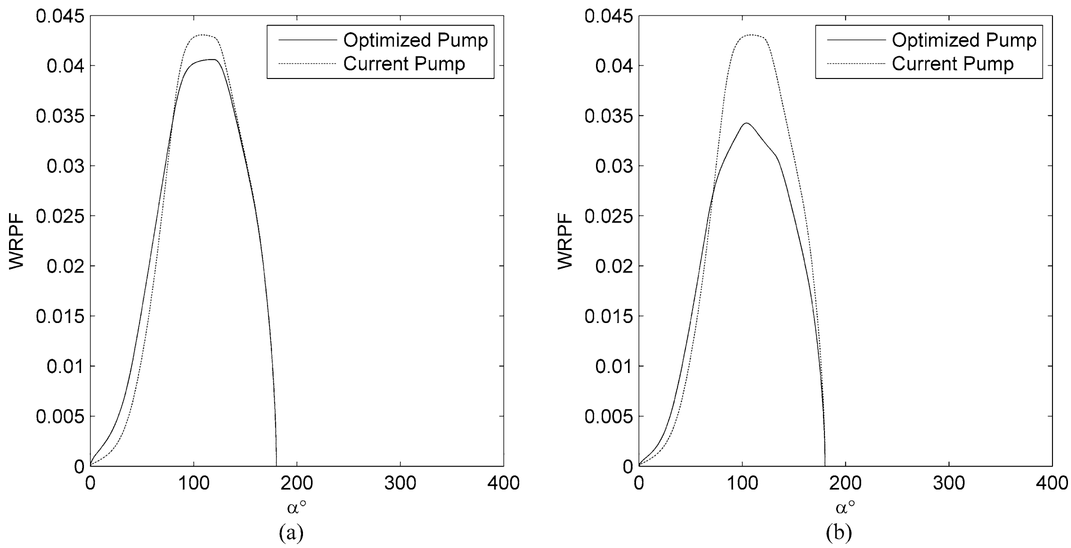

Fig. 9 "Wear Rate Proportional Factor" diagrams for Case 1 and its optimum profiles with (**a**) $N = 9$, (**b**) $N = 11$

Table 7 Geometrical parameters of two commercial pumps

	N	a (mm)	R (mm)	e (mm)	H (mm)	FR (%)	WRPF
Case 1 [6]	9	40.725	10.85	2.85	9.25	2.5	0.0431
Case 2 (PZ9e19) [7]	9	22.4	4.55	1.9	8.895	2.6	0.0438

Table 8 The optimum results of the two commercial pumps

	N	a (mm)	R (mm)	e (mm)	H (mm)	FR (%)	FR improvement	WRPF	WRPF improvement
Case 1	9	40.725	8.3835	2.6377	9.25	2.09	16.4%	0.0406	5.81%
	11	40.725	6.3362	2.4499	9.25	1.35	46%	0.0343	20.41%
Case 2	9	22.4	3.837	1.8307	8.895	2.25	14%	0.0423	3.42%
	11	22.4	2.9221	1.7318	8.895	1.61	38.08%	0.0365	16.67%

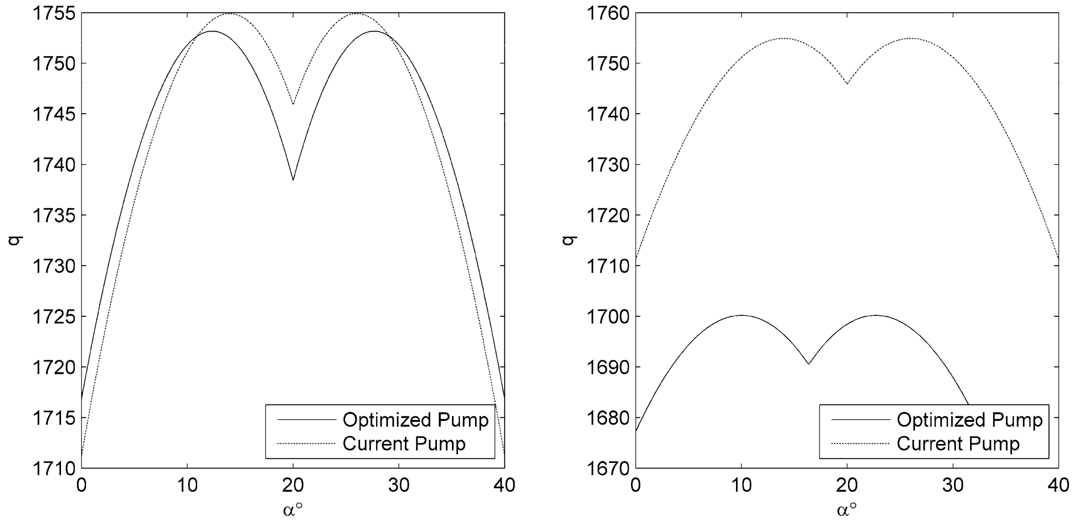

Fig. 10 "Flow rate" diagrams for Case 1 and its optimum profiles with (**a**) $N = 9$, (**b**) $N = 11$

Table 6 Critical value of number of outer rotor teeth in relation to WRPF

q_w	0.2	0.4	0.8	1	1.6	2.2
N_{cr}	32	28	13	10	6	4

straints have been formulated too. The "Global Criterion Method" has been used to change the multi objective optimization problem to single objective one and the "Line-up Competition Algorithm" method has been used to solve the single objective optimization problem. The optimum solution has been found for several values of non-dimensional displacement and rotor width. The results show the pumps with odd values of number of outer rotor teeth have smaller flow irregularity. Also greater value of N causes better flow irregularity. The WRPF is decreased as teeth number is increased while teeth number reaches a critical value. Also these critical values of N decrease by increasing of non-dimensional displacement. So greater value of a can be useful if it is possible, because greater value of a causes smaller q_w. As N is increased the ability of the Gerotor pump is decreased for providing of a specific displacement. It is seen that as x_{3max} increases flow irregularity increase for even value of N while it decrease for odd value of N. Also WRPF is decreased by increasing of x_{3max} for both odd and even value of N. So larger value of x_{3max} is useful for a pump with odd number of outer rotor teeth but x_{3max} value could be chosen according to the importance of each cost function and the requirement non-dimensional displacement for a pump with even value of N.

At last two commercial pumps have been optimized and the value of the cost functions has been compared with those of the current pump. These comparisons show the improvement of both flow irregularity and WRPF of both pumps. So the present optimization approach can be used to optimum design of the Gerotor pumps. Also it proposes a pattern for choosing of the design variables according to space limitations.

Considering viscose contact, input torque as a function of input and output pressure, both Epitrochoidal and Hypotrochoidal profiles and etc. could be attended as future works.

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