Aerodynamics

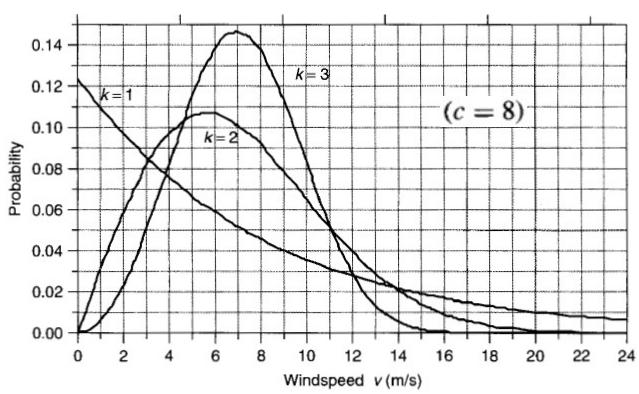
The aerodynamic concepts of wind turbines

Weibull Probability Distribution function

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad \text{Weibull p.d.f.}$$

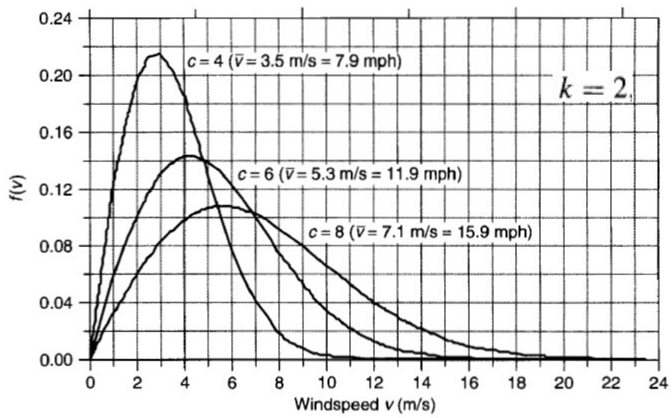
c: scale parameter

k: shape parameter



Wind Speed Distribution

$$f(v) = \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right] \qquad \text{Rayleigh p.d.f.}$$



Average Wind Speed

$$\bar{v} = \int_0^\infty v \cdot f(v)\, dv = \int_0^\infty \frac{2v^2}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right]$$

Using:

$$\Gamma(n+1) = \int_0^\infty e^{-x} x^{n+1-1} dx = \int_0^\infty e^{-x} x^n dx$$

We have: $\bar{V} = c\Gamma\left(1+\frac{1}{2}\right) = c\Gamma\left(\frac{3}{2}\right)$ and $\Gamma(3/2) = \frac{\sqrt{\pi}}{2} \approx 0.886$

So: $\bar{V} = c\frac{\sqrt{\pi}}{2} = 0.886c \cong 0.9c$ ⇒ $c \cong \bar{V}/0.9$

⇒ $f(v) = \frac{\pi v}{2\bar{v}^2} \exp\left[-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2\right]$ Rayleigh

Wind Power Classes

Estimates of the wind resource are expressed in wind power classes ranging from Class 1 to Class 7, with each class representing a range of mean wind power density or equivalent mean speed at specified heights above the ground. This map does not show Classes 1 and 2 as Class 2 areas are marginal and Class 1 areas are unsuitable for utility-scale wind energy development.

Wind Power Classification

Wind Power Class	Resource Potential	Wind Power Density at 50 m W/m^2	Wind Speed[a] at 50 m m/s	Wind Speed[a] at 50 m mph
3	Fair	300 - 400	6.4 - 7.0	14.3 - 15.7
4	Good	400 - 500	7.0 - 7.5	15.7 - 16.8
5	Excellent	500 - 600	7.5 - 8.0	16.8 - 17.9
6	Outstanding	600 - 800	8.0 - 8.8	17.9 - 19.7
7	Superb	800 - 1600	8.8 - 11.1	19.7 - 24.8

[a] Wind speeds are based on a Weibull k value of 2.0

Wind Power Class	Resource Potential	Wind Speed at 50 m (mph)
Class 1	Poor	0.0 - 12.5
Class 2	Marginal	12.5 - 14.3
Class 3	Fair	14.3 - 15.7
Class 4	Good	15.7 - 16.8
Class 5	Excellent	16.8 - 17.9
Class 6	Outstanding	17.9 - 19.7
Class 7	Superb	above 19.7

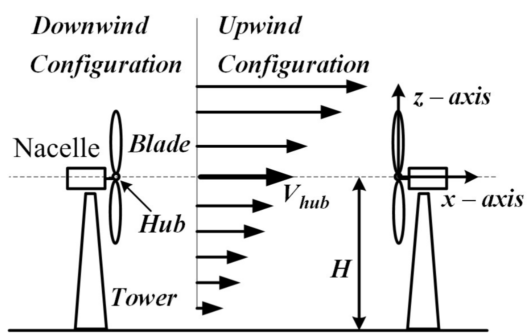
Standard Wind Class Definitions

Class	30 m height		50m height	
	Wind speed m/s	Wind power W/m ^{2}	Wind speed m/s	Wind power W/m ^{2}
1	0-5.1	0-160	0-5.6	0-200
2	5.1-5.9	160-240	5.6-6.4	200-300
3	5.9-6.5	240-320	6.4-7.0	300-400
4	6.5-7.0	320-400	7.0-7.5	400-500
5	7.0-7.4	400-480	7.5-8.0	500-600
6	7.4-8.2	480-640	8.0-8.8	600-800
7	8.2-11.0	640-1600	8.8-11.9	800-2000

Aerodynamic Aspects of Wind Energy Conversion System

- Tower Wake (Shadow) Effect
- Wind Shears
  - Vertical wind shear
  - Horizontal wind shear
- Yaw Error

Vertical Wind Shear

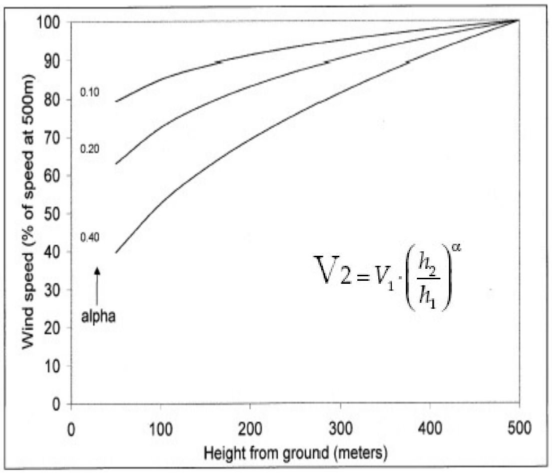


$V(z) \propto (\frac{z}{H})^{V_{shr}}$

H: hub height, V_{shr}: wind shear exponent,
$V(z)$: wind velocity at z

Source: www.windpower.org

Changes in speed with height and friction



Friction Coefficient of Various Terrain

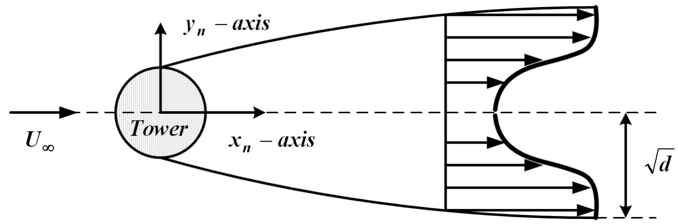
Terrain Type	Friction Coefficient α
Lake, ocean and smooth hard ground	0.10
Foot high grass on level ground	0.15
Tall crops, hedges, and shrubs	0.20
Wooded country with many trees	0.25
Small town with some trees and shrubs	0.30
City area with tall buildings	0.40

Roughness Length

Roughness Length m*	Wind Speed Increase from 20m to 80m	Wind Speeds at 80m	Energy Increase 20m to 80m	1000 kw Turbine Capacity Factor at 80m	100 kw Turbine Project Capacity Increase for Equivalent Energy Yield	Landscape Type
0.0002	12%	5.60	35%	18.8%	158%	Water surface
0.0024	15%	5.77	39%	20.2%	170%	Completely open terrain with a smooth surface, e.g.concrete runways in airports, mowed grass, etc.
0.03	21%	6.07	46%	22.6%	190%	Open agricultural area without fences and hedgerows and very scattered buildings. Only softly rounded hills
0.055	24%	6.18	48%	23.5%	197%	Agricultural land with some houses and 8 metre tall sheltering hedgerows with a distance of approx. 1250 metres
0.1	26%	6.31	51%	24.5%	206%	Agricultural land with some houses and 8 metre tall sheltering hedgerows with a distance of approx. 500 metres
0.2	30%	6.51	55%	26.1%	219%	Agricultural land with many houses, shrubs and plants, or 8 metre tall sheltering hedgerows with a distance of approx. 250 metres
0.4	35%	6.77	60%	28.2%	237%	Villages, small towns, agricultural land with many or tall sheltering hedgerows, forests and very rough and uneven terrain

$$V = Vref \frac{Ln(z / z_0)}{Ln(z_{ref} / z_0)}$$

Tower Shadow (Wake Effect)



$$U_{local} = (1 - u_{wake})U_\infty$$

$$u_{wake} = \begin{cases} \frac{C_d}{\sqrt{d}} cos^2(\frac{\pi}{2}\frac{y_n}{\sqrt{d}}) & \text{for } |y_n| \le \sqrt{d} \\ 0 & \text{for } |y_n| > \sqrt{d} \end{cases}$$

$$d = \sqrt{x_n^2 + y_n^2}$$

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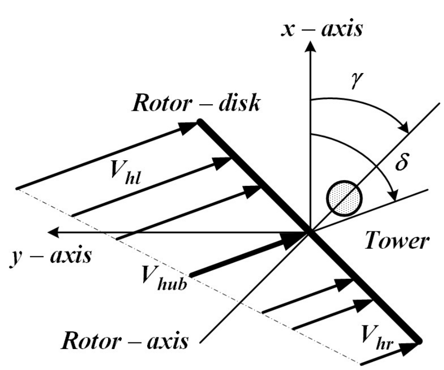
Horizontal Wind Shear

$H_{shr} = \frac{V_{hl} - V_{hr}}{V_{hub}}$

H_{shr}: horizontal wind shear,

V_{hub}:hub-height wind speed



Top view

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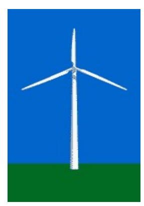
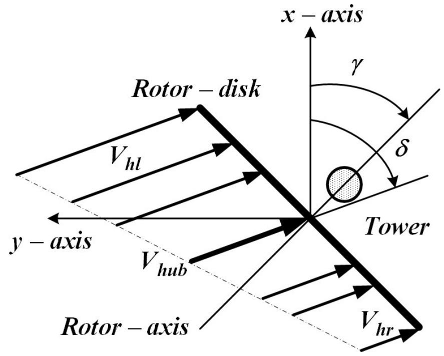
Yaw Error

$$V_1 = V_{hub}\left[(1+\frac{z}{H})^{V_{shr}} + \frac{H_{shr}(ycos(\delta-\gamma))}{2R} + \frac{H_{shr}(xsin(\delta-\gamma)) + zV_{shr_{lin}}}{2R}\right] + V_{gust}$$

$$V_x = V_1 cos(\delta-\gamma)$$
$$V_y = V_1 sin(\delta-\gamma)$$

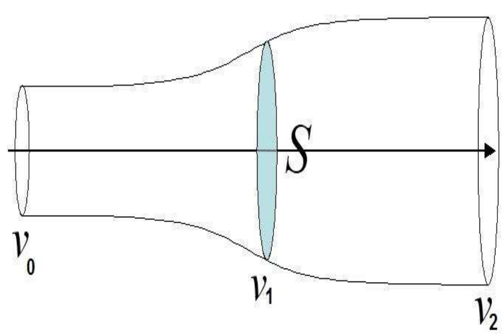
$\delta-\gamma$: The yaw error angle,





C_P=4a(1-a)^2

Extracted Power

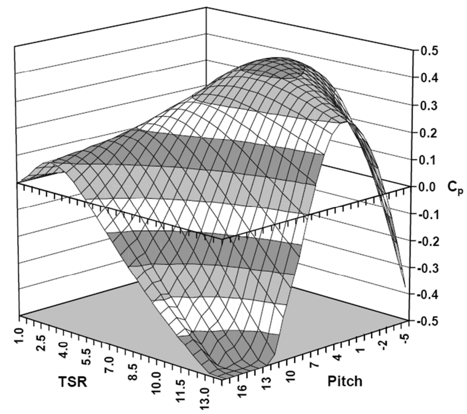


$P = C_P P_0$ $\quad P_0 = \frac{1}{2}\rho A_1 V_0^3$ $\quad C_P = 4a(1-a)^2$

Aerodynamic Conversion

$P = 0.5 \rho A C_P V^3$
A:Area of the rotor disk,
ρ:Density of air,
V:Wind velocity,
C_P:Power coefficient,

$\lambda = \frac{\Omega r}{V}$:Tip speed ratio,
Ω:Rotor speed,
r:Rotor radius,
β:The blade pitch angle



The C_P verses λ and β

Source: NREL technical report

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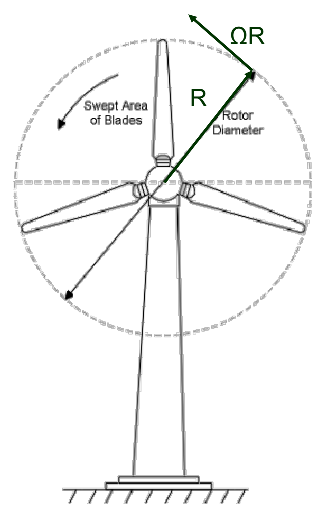
Tip Speed Ratio

$\lambda = \frac{\Omega R}{V}$:Tip speed ratio,
Ω:Rotor speed,
R:Rotor radius,
V:Wind velocity,



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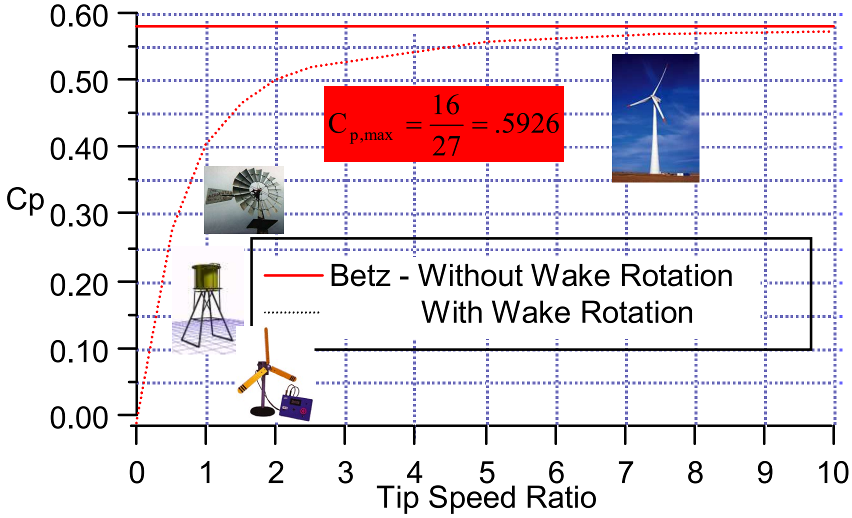
Maximum Possible Power Coefficient



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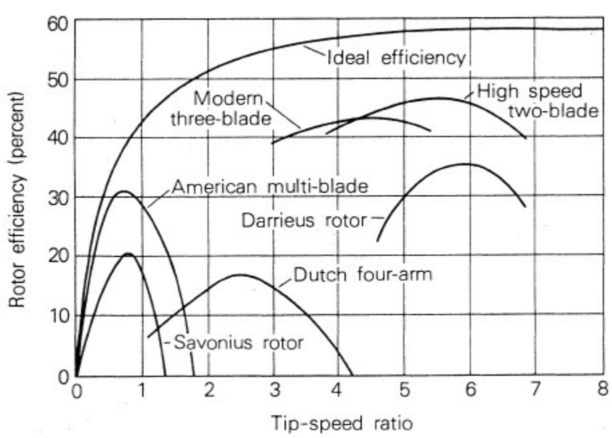
WTs' efficiencies



Air Density Variations

$$\rho = \frac{p}{R \cdot T}$$

where p = air pressure
T = temperature on the absolute scale
R = gas constant.

With elevation:

$$\rho = \rho_o \cdot e^{-\left\{\frac{0.297\,H_m}{3048}\right\}}$$

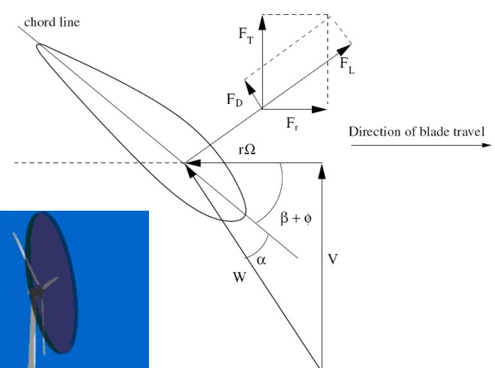
Where: r_0=1.225 kg/m^3

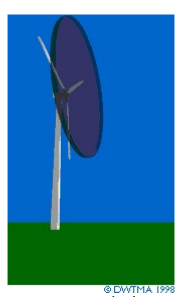
where H_m is the site elevation in meters

Or: $\rho = \rho_o - 1.194 \cdot 10^{-4} \cdot H_m$ and $T = 15.5 - \frac{19.83\,H_m}{3048}$ °C

Aerodynamic Conversion (cont.)

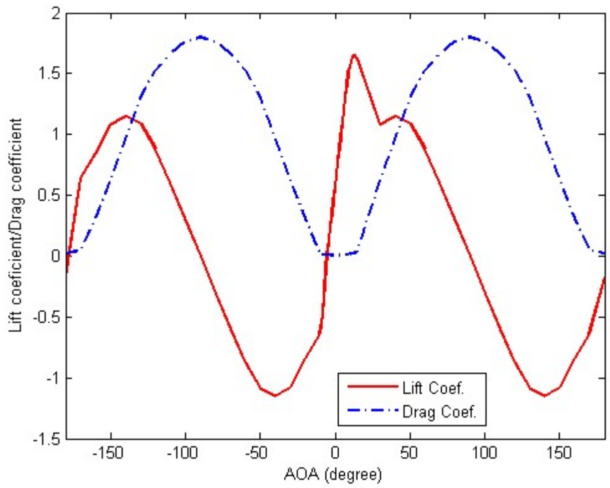
$F_L = 0.5\rho c C_L W^2$
$F_D = 0.5\rho c C_D W^2$
F_L:lift force,
F_D:drag force,
ρ:density of air,
c:chord length,
C_L:lift coefficient,
C_D:drag coefficient
W:relative velocity,
V:wind velocity
α:angle of attack
β:blade pitch angle
ϕ:blade twist angle





Source: www.windpower.org

The Lift/Drag Coefficient for Aerodynamic forces calculations



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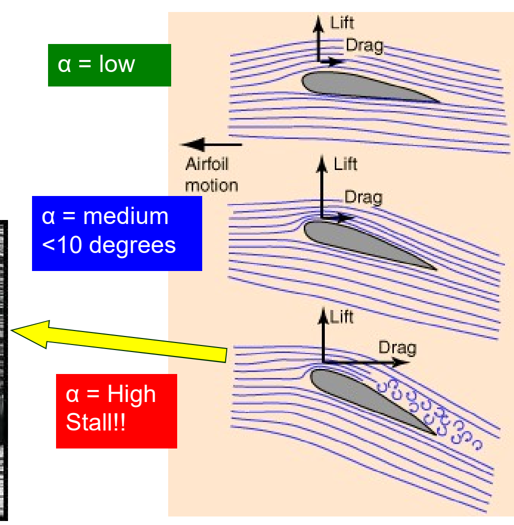
Airfoil Behavior



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