Calculating Solar Angles

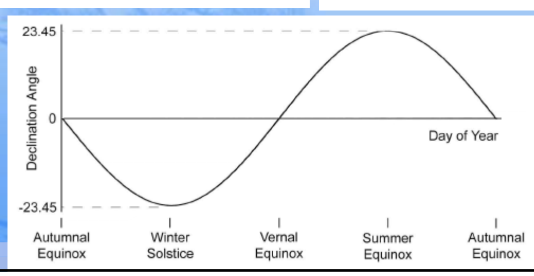
(Angles in Radian)

Declination Angle

- The equation used to calculate the declination angle in radians on any given day is:

$$\delta = 23.45\frac{\pi}{180}\sin\left[2\pi\left(\frac{284+n}{365.25}\right)\right]$$

where:
δ = declination angle (rads);
n = the day number, such that n = 1 on the 1^{st} January.



The hour angle

- The hour angle is positive during the morning, reduces to zero at solar noon and becomes increasingly negative as the afternoon progresses.
- Two equations can be used to calculate the hour angle when various angles are known:

$$\sin\omega = -\frac{\cos\alpha \sin A_Z}{\cos\delta}$$

$$\sin\omega = \frac{\sin\alpha - \sin\delta \sin\phi}{\cos\delta \cos\phi}$$

ω = the hour angle;
α = the altitude angle;

A_Z= the solar azimuth angle;
δ = the declination angle;
ϕ = observer's latitude.

The hour angle

- At solar noon, the hour angle equals zero
- Since the hour angle changes at 15° per hour it is a simple matter to calculate the hour angle at any time of day.
- The hour angles at sunrise and sunset are very useful quantities to know.
- Before finding them, altitude angle relation is discussed:

The altitude angle

- Solar altitude angle is the angle between the horizontal and the line to the sun, i.e., the complement of the zenith angle.
- It can be calculated from:

$$\sin\alpha = \sin\delta\sin\phi + \cos\delta\cos\omega\cos\phi$$

Declination angle (δ), Latitude angle (ϕ), Hour angle (ω)

Hour angle at sunrise and sunset

- Substituting α=0 gives:

$$\cos\omega_s = -\tan\phi\ \tan\delta$$

- This equation gives hour angle at sunrise (negative) or sunset (positive).
- The number of daylight hour can be found:

$$N = \frac{2\omega_s}{15}\times\frac{180}{\pi}$$

Solar azimuth angle

- Solar azimuth angle is the angular displacement from south of the projection of beam radiation on the horizontal plane. Displacement east of south are negative.

$$\sin\alpha = \frac{\sin\omega\cos\delta}{\sin\theta_z} = \frac{\sin\omega\cos\delta}{\cos\alpha}$$

Solar zenith angle

Angle of incidence (θ_i)

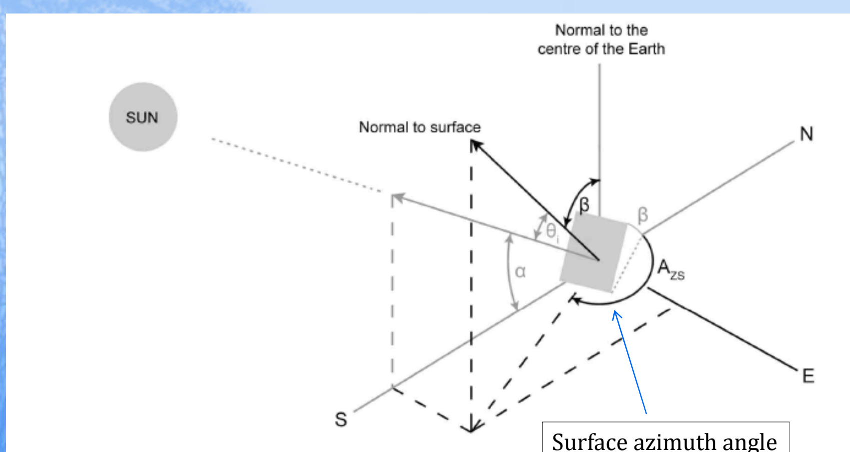


Figure 3.2: A tilted surface that is not facing the equator.

Calculation of solar incident angle

The angle of incidence (θ_i) of the Sun on a surface tilted at an angle from the horizontal (β) and with any surface azimuth angle (A_{ZS}) (figure 3.2) can be calculated from (when A_{ZS} is measured clockwise from north):

$$\cos\theta_i = \sin\delta\sin\phi\cos\beta + \sin\delta\cos\phi\sin\beta\cos A_{ZS} + \cos\delta\cos\phi\cos\beta\cos\omega - \cos\delta\sin\phi\sin\beta\cos A_{ZS}\cos\omega - \cos\delta\sin\beta\sin A_{ZS}\sin\omega \quad (3.11)$$

This horrible equation can be simplified in a number of instances. When the surface is flat (i.e. horizontal) $\beta = 0$, $\cos\beta = 1$, $\sin\beta = 0$. Therefore equation 3.11 becomes:

$$\cos\theta_i = \cos\theta_z = \cos\delta\cos\phi\ \cos\omega + \sin\delta\sin\phi \quad (3.12)$$

When the surface is tilted towards the equator (facing south in the northern hemisphere):

$$\cos\theta_i = \cos\delta\cos(\phi - \beta)\cos\omega + \sin\delta\sin(\phi - \beta) \quad (3.13)$$

Note that if $\theta_i > 90°$ at any point the Sun is behind the surface and the surface will be shading itself.

Ratio of beam radiation on a titled surface to that on horizontal surface

Geometric factor

- Common available data are total radiation for hours or days on horizontal surface, whereas the need is for beam and diffuse radiation on the plane of a collector.
- The geometric factor R_b, is the ratio of beam radiation on the tilted surface to that on a horizontal surface:

$$R_b = \frac{G_{b,T}}{G_b} = \frac{G_{b,n}\cos\theta}{G_{b,n}\cos\theta_z} = \frac{\cos\theta}{\cos\theta_z}$$

Geometric factor

- The optimum azimuth angle for flat-plate collectors is usually 0°in the northern hemisphere and 180° for southern hemisphere. In this case, substitution of equations for cosθ and $\cos\theta_z$ leads to the following equation for northern hemisphere:

$$R_b = \frac{\cos(\phi-\beta)\cos\delta\cos\omega + \sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\cos\omega + \sin\phi\sin\delta}$$